

# Math 3201 - Sample Public Exam

- |       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 1. A  | 11. A | 21. C | 31. D | 41. A |
| 2. D  | 12. A | 22. D | 32. A | 42. D |
| 3. B  | 13. C | 23. A | 33. D | 43. B |
| 4. B  | 14. B | 24. A | 34. D | 44. B |
| 5. C  | 15. C | 25. D | 35. A | 45. D |
| 6. D  | 16. C | 26. C | 36. B | 46. B |
| 7. B  | 17. C | 27. A | 37. A | 47. B |
| 8. A  | 18. D | 28. D | 38. B | 48. D |
| 9. A  | 19. B | 29. D | 39. C | 49. B |
| 10. B | 20. A | 30. B | 40. B | 50. A |

51.  $x=6$        $6x=36\%$   
 $36\%$  of  $200 = 72$ .

52(a)  $2! \cdot 5! = 240$

52(b)  $n = -3$  reject  
 $n = 4$

52(c)  ${}_{12}P_4 = 11,880$

53(a)  $\frac{48}{51} = 40\%$

53(b)  $\frac{3}{4} \times \frac{8}{15} = 40\%$

54(a)  $\frac{3x-1}{12}$ ,  $x \neq \pm 1, \frac{1}{3}$

54(b)  $\frac{1}{2x} + \frac{1}{x} = \frac{1}{5}$   
 $x = 7.5$



Different # of x-intercepts

55(b)(i)  $y$ -int =  $-4$

end beh. Q III to Q I

Max # 3

(ii) Degree is 3, not 2.

56(a)  $x = -5/24$

56(b)  $A = 5000(1+0.005)^2$   
 and  $A = 5000(1+\frac{0.65}{2})^1$

Invest in the financial institution.

57(a)  $x = -7.85$

57(b)(i)  $x = 0.00001 \text{ mol/L}$

(ii)  $\frac{10^{-5}}{10^{-9}} = 10,000$  times more acidic

58(a)(i) amp = 2  
eqn.  $y = -1$   
period =  $120^\circ$   
range  $-3 \leq y \leq 1$

(ii)  $y = 2 \cos 3(x) - 1$

59.  $A = \$3175.59$

interest  $\rightarrow \$675.59$

PART II  
Total Value: 50%

Answer ALL items in the space provided. Show ALL workings.

Value

- 3 51. 200 students wrote exams in Math, Biology and English. The Venn Diagram below represents the percentage of those who wrote the exams. Algebraically determine the percentage of students who wrote all three exams, and determine the number of students that this represents.

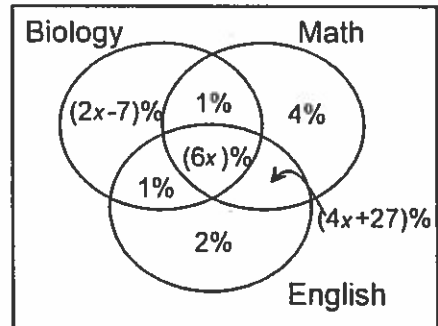
$$2x - 7 + 6x + 8 + 4x + 27 = 100$$

$$12x + 28 = 100$$

$$12x = 100 - 28$$

$$\frac{12x}{12} = \frac{72}{12}$$

$$x = 6$$



$$\begin{aligned} (6x)\% &= (6(6))\% \\ &= 36\% \end{aligned}$$

$$\begin{aligned} &36\% \text{ of } 200 \\ &= 0.36(200) \\ &= 72. \end{aligned}$$

- 2 52.(a) In how many ways can a teacher seat four girls and two boys in a row of six seats if the two boys must be seated next to each other?

$$\begin{aligned} 2! \cdot 5! &= 2 \times 120 \\ &= 240. \end{aligned}$$

Value

3 52.(b) Algebraically solve for  $n$ :  ${}_nP_2 = 12$

$$\frac{n!}{(n-2)!} = 12$$

$$\frac{n \cdot (n-1) \cdot \cancel{(n-2)!}}{\cancel{(n-2)!}} = 12$$

$$n(n-1) = 12$$

$$n^2 - n - 12 = 0$$

$$(n-4)(n+3) = 0$$

$$\textcircled{n=4} \quad n = -3 \text{ reject.}$$

2 52.(c) Four students are to be chosen from a group of 12 to fill the positions of president, vice-president, treasurer and secretary. In how many ways can this be accomplished?

$$12 \times 11 \times 10 \times 9 = 11,880$$

or

$${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 11,880.$$

3 53.(a) If a 5-digit number is generated at random from the digits 2, 3, 4, 5 and 8 (with no repetition), what is the probability that it will be an odd number?

$$p(\text{odd}) = \frac{\# \text{ odd}}{\text{total}} = \frac{48}{5!} = \frac{48}{120} = 40\%.$$

$$4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 = 48$$

$$5! = 120$$

Value

- 3 53.(b) A person will be randomly selected from a group to draw a marble from a bag. The odds of selecting a female from the group is 7:8 and the odds of drawing a red marble from the bag are 1:3. What is the probability of a non-red marble being drawn from the bag by a male from the group?

$$F : M$$

$$7 : 8$$

R

$$1 : 3$$

$$P(R') \times P(M) = \frac{2}{4} \times \frac{8}{15} = \frac{2}{5} = 0.4 = 40\%$$

- 4 54.(a) Simplify and state restrictions:  $\frac{1-x^2}{8-8x} \div \frac{3x+3}{2(3x-1)}$

$$\begin{aligned} & \frac{-1(x^2-1)}{-1(8x-8)} \cdot \frac{2(3x-1)}{3(x+1)} \\ & = \frac{\cancel{-1}(x+1)\cancel{(x-1)}}{\cancel{-1}(\cancel{8}x-\cancel{8})} \cdot \frac{2(3x-1)}{3\cancel{(x+1)}} \\ & = \frac{(3x-1)}{12} \end{aligned}$$

$$x \neq 1, -1, \frac{1}{3}$$

Value

2

54.(b) Pat and Chris can paint the house in 5 hours if they work together. Pat is a professional painter and can paint twice as fast as Chris. How long would it take Pat to paint the house by himself?

Pat	$x$	$\frac{1}{x}$
Chris	$2x$	$\frac{1}{2x}$
Together	$5$	$\frac{1}{5}$

$$\left( \frac{1}{2x} + \frac{1}{x} = \frac{1}{5} \right) (10x)$$

$$5 + 10 = 2x$$

$$15 = \frac{2x}{2}$$

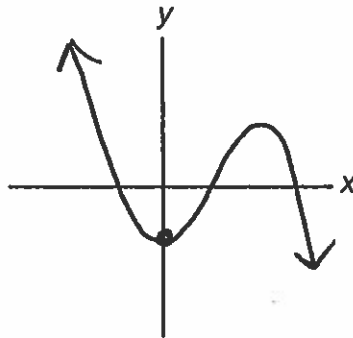
$$x = 7.5$$

Takes Pat  
 $7\frac{1}{2}$  hrs by  
himself.

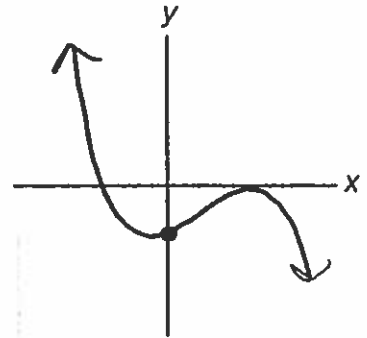
2

55.(a) Sketch two possible graphs that are different, yet are both cubic functions with negative leading coefficients and negative y-intercepts. Explain why the graphs you have sketched are different.

Graph 1:



Graph 2:



Different number of x-intercepts.

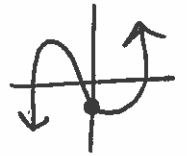
Value

4 55.(b) Given the function  $f(x) = 2x^3 + 5x^2 - 3x - 4$ , complete the table to describe its characteristics.

(3)

(i)

y-intercept	let $x=0$ $f(0) = -4$
end behaviour (left and right)	QIII to QI
Max # of possible x-intercepts	3



(1)

(ii) Explain why the graph of this function is not a parabola.

Degree is 3, not 2.

3

56.(a) Algebraically solve for x:  $\sqrt{3} = 27^{4x+1}$

$$3^{1/2} = 3^{3(4x+1)}$$

$$3^{1/2} = 3^{12x+3}$$

$$\frac{1}{2} = 12x + 3$$

$$1 = 24x + 6$$

$$1 - 6 = 24x$$

$$-5 = 24x$$

$$\frac{-5}{24} = \frac{24x}{24}$$

$$x = -\frac{5}{24}$$

Value

4

56.(b) Nora is about to invest \$5000 in an account that pays 6% interest a year compounded monthly for the next 3 years. A different financial institution offers 6.5% interest a year compounded semi-annually for the next 3 years. Write a function that models the growth of Nora's investment for each situation. Should Nora invest her money in this financial institution instead? Explain why or why not.

$$P = 5000$$

$$i = \frac{6\%}{12} = 0.005$$

$$n = 3(12) = 36$$

$$A = 5000(1 + 0.005)^{36}$$

$$A = 5983.40$$

$$A = 5000\left(1 + \frac{0.065}{2}\right)^6$$

$$A = 6057.74$$

She should invest in the financial institution.

$$i = \frac{6.5\%}{2} = 0.0325$$

$$n = 3(2) = 6$$

4

57.(a) Algebraically solve for x:  $5^{x-1} - 8^{x+1} = 0$

$$5^{x-1} = 8^{x+1}$$

$$(x-1)\log 5 = (x+1)\log 8$$

$$x\log 5 - \log 5 = x\log 8 + \log 8$$

$$x\log 5 - x\log 8 = \log 8 + \log 5$$

$$x \frac{(\log 5 - \log 8)}{\log 5 - \log 8} = \frac{\log 8 + \log 5}{\log 5 - \log 8}$$

$$x = \frac{\log 8 + \log 5}{\log 5 - \log 8}$$

$$x = -7.85$$



Value

3 57.(b) The pH scale is used to measure the acidity of a solution. The pH,  $p(x)$ , is defined by the equation  $p(x) = -\log x$ , where the concentration of hydrogen ions,  $x$ , in a solution is measured in moles per litre (mol/L).

(1) (i) Black coffee has a pH of 5. What is its hydrogen ion concentration?

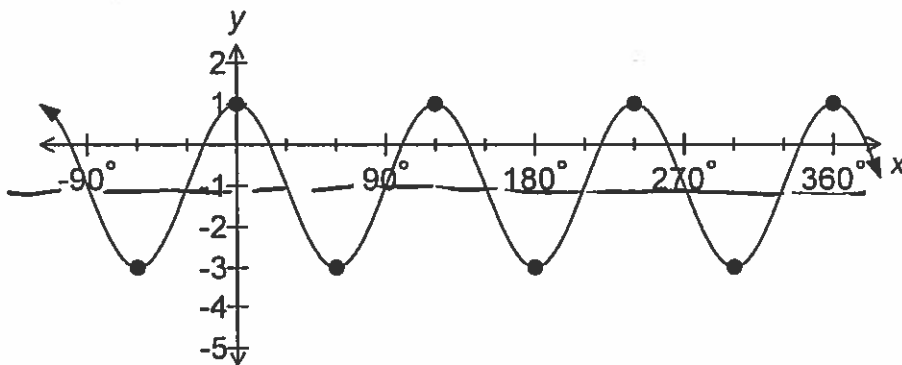
$$\begin{aligned} 5 &= -\log x \\ -5 &= \log x \\ 10^{-5} &= x \\ x &= 0.00001 \text{ mol/L} \end{aligned}$$

(2) (ii) Baking soda has a pH of 9. In terms of concentration, how much more acidic is black coffee than baking soda?

Coffee	Baking Soda
$10^{-5}$	$10^{-9}$

$$\frac{10^{-5}}{10^{-9}} = 10^4 = 10,000 \text{ times more acidic.}$$

6 58.(a) Use the sinusoidal function shown below to answer the questions that follow.



(4) (i) Determine the amplitude, period, equation of midline and the range.

$$\begin{aligned} \text{eqn. of midline } &y = -1 \\ \text{amp} &= 2 \\ \text{period} &= 120^\circ \\ \text{range } &-3 \leq y \leq 1 \end{aligned}$$

(2) (ii) Use the information from part (i) to determine a function that represents the graph in the form  $y = a \cos b(x) + d$ .

$$\begin{aligned} 120^\circ &= \frac{360^\circ}{b} \\ b(120^\circ) &= \frac{360^\circ}{120^\circ} \\ b &= 3 \end{aligned}$$

$$y = 2 \cos 3(x) - 1$$

Value

- 3 59. Pat borrowed \$2500 at a rate of 8% compounded monthly for 3 years. How much interest will Pat be charged for borrowing the money?

$$P = 2500$$

$$i = \frac{8\%}{12} = \frac{0.08}{12}$$

$$n = 3(12) \\ = 36$$

$$A = 2500 \left(1 + \frac{0.08}{12}\right)^{36}$$

$$A = \$3175.59$$

$$3175.59$$

$$- 2500$$

$$\underline{\$675.59} \rightarrow \text{interest}$$