Section 2.3: Permutations When All Objects Are Distinguishable

Permutation

→ ***arranging all or part of a set of distinguishable objects where ORDER is IMPORTANT***

What are the possible permutations of the letters A, B and C ?

ABC, ACB, BAC, BCA, CAB, CBA

Example 1:

How many ways are there to arrange 3 people of a group of 5 in a line?

FCP: ____ ____ ____

This expression can be written as: $\frac{5!}{2!} = \frac{5!}{(5-3)!} = {}_5P_3$

Formula to determine the number of permutations of ***n*** different elements taken ***r*** at a time:

$${}_nP_r = \frac{n!}{(n-r)!}$$

- arranging a subset of items
- only some of the items are used in the arrangement

OR ${}_nP_n$ if all objects are used!

→

Example 2:

If there are 7 members on the student council, how many ways can the council select 3 students to be the president, vice-president and the treasurer?

Example 3:

In how many ways can 6 people be arranged in a line for a photograph?

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!}$$

⟵ #ways to count an empty set

$$0! = 1$$

Example 4:

A code consists of three letters chosen from A to Z and three digits chosen from 0 to 9, with no repetitions of letters or numbers. Determine the total number of possible codes.

⟶

Permutations

↳ With Constraints

- two or more objects must be placed together
- two or more objects cannot be placed together
- certain objects must be placed in certain positions

Example 5:

a) How many ways can a group of 5 people be arranged in a line if two of them are good friends, Mary and Sam, and want to sit together.

Mary Sam
Dave
Kyle
John

★ when certain items are to be kept together, treat the joined item as if they were only one object.

b) How many ways can a group of 5 people be arranged in a line if Mary and Sam should <u>not</u> sit together.

Complement

total # of arrangements with no restrictions — arrangements with Mary and Sam together

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Example 6:

At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a) The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

b) The three red cars must be parked side by side. How many ways can the seven cars be parked?

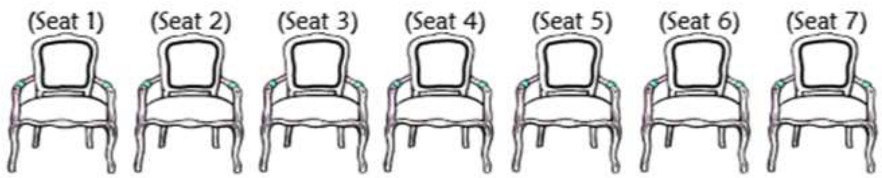
Arrangements Involving Cases

→ Some problems have more than one case. Calculate the number of arrangements for each case and then **add** up the values for all cases to obtain the total.

Key Words: At Least, At Most , Either

Example 7:

Determine the number of arrangements of 4 girls and 3 boys in a row of seven seats if the ends of the rows must be <u>either</u> both female <u>or</u> both male.



4 girls, 3 boys

Case for Females	Case for Males

Example 8: (Ex. 3, p. 88)

Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and/or any letters of the alphabet. The password is ***case sensitive***, so she can use both lower and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

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Example 9:

A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if each digit can be repeated? How does this compare with the number of SINs that can be created if no repetition is allowed?

Example 10:

Solve equations using ${}_nP_r$ where $n > 0,\ n > r$

a) Solve: ${}_nP_2 = 30$

b) Solve: ${}_{n-1}P_2 = 12$

Example 11:

Mary has a set of posters to arrange on her bedroom wall. she can only fit 2 posters side by side. If there are 72 ways to choose and arrange 2 posters, how many posters does she have in total?

Practice Questions:

p.93-94, #1acde,3,5,8,9,10abc,13ab,14ab,15ab