Section 3.4: Mutually Exclusive Events

Recall from Unit 1 we classified events as:

mutually exclusive (disjoint sets) and non-mutually exclusive.

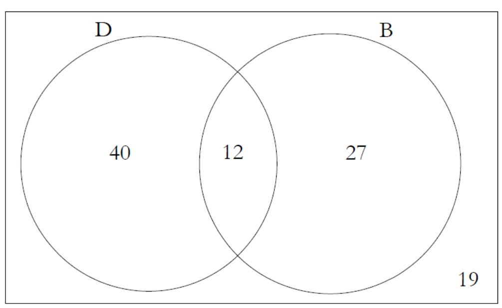
Mutually exclusive sets did not intersect.

(i.e, $n(A \cap B) = 0$)

Recall also that the outcomes of an event and the outcomes of the complement make up the entire sample space.

We will now solve probability problems involving mutually exclusive and non-mutually exclusive events.

Let's look at an example of a Venn diagram where D represents students on the debate team, and B represents students on the Basketball team.



In order to determine the probability of events, we have to think about the following questions:

- **Are the two sets intersecting or disjoint?**
- **How many elements are in each set?**
- **How many elements are in the universal set** *S*?

Using the Principle of Inclusion and Exclusion, we can develop the probability formula for non-mutually exclusive events:

$$n(D \cup B) = n(D) + n(B) - n(D \cap B)$$

$$P(D \cup B) = \frac{n(D) + n(B) - n(D \cap B)}{n(S)}$$

$$P(D \cup B) = \frac{n(D)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(D \cap B)}{n(S)}$$

$$P(D \cup B) = P(D) + P(B) - P(D \cap B)$$

Recall if events are mutually exclusive, the sets are disjoint, so $n(D \cap B) = 0$.

Therefore, $P(D \cup B) = P(D) + P(B)$

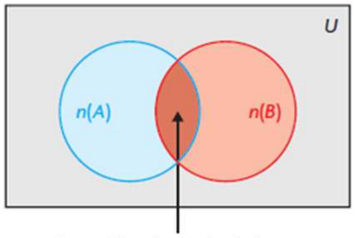
To summarize:

If A and B are non-mutually exclusive events,

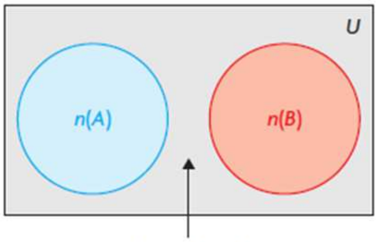
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B)$$



n(A ∩ B) has been shaded twice



n(A ∩ B) = 0
(no common elements)

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Example 1:

Classify the events in each experiment as either mutually exclusive or non-mutually exclusive:

a) The experiment is rolling a die. The first event is rolling an even number and the second event is rolling a prime number.

b) The experiment is playing a game of hockey. The first event is that your team scores a goal, and the second event is that your team wins the game.

c) The experiment is selecting a gift. The first event is that the gift is edible and the second event is that the gift is an iPhone.

NOTE:

The sum of the probability of an event and its complement must equal 1.

$$P(A) + P(A') = 1$$

Rearranging the formula would give us:

$$P(A') = 1 - P(A) \quad \text{and} \quad P(A) = 1 - P(A')$$

For example,

If the probability that a student picks the ace of diamonds from a standard deck of cards is: $\frac{1}{52}$

then the probability that he/she will not pick the ace of diamonds is: $\frac{51}{52}$ $\left(\frac{1}{52} + \frac{51}{52} = 1\right)$

Example 2:

Determine if the events are mutually exclusive or non-mutually exclusive and display the information in a Venn diagram.

Class Survey

63% of students play sports

27% of the students play a musical instrument

20% play neither sports nor a musical instrument

Example 3: (ex. 2, p. 168)

Jack and Ellen are playing a board game. If a player rolls a sum that is greater than 8 or a multiple of 5 when using 2 standard dice, the player gets a bonus of 100 points. Determine the probability that Ellen will receive a bonus of 100 points on her next roll.

Possible Sums When a Pair of Dice are Rolled

Die 1/ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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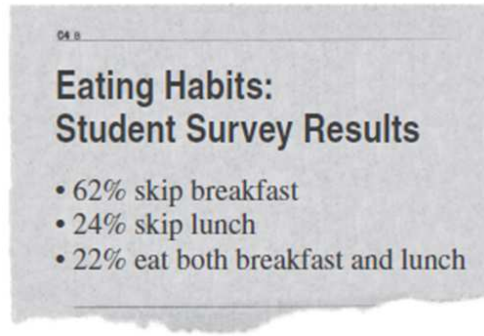
Example 4: (ex. 3, p. 170)

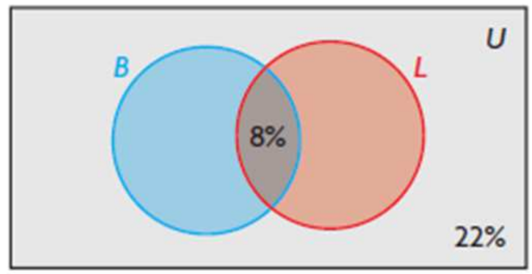
A school newspaper published the results of a recent survey.



Eating Habits: Student Survey Results

- 62% skip breakfast
- 24% skip lunch
- 22% eat both breakfast and lunch

a) Are skipping breakfast and skipping lunch mutually exclusive events?



b) Determine the probability that a randomly selected student skips breakfast but not lunch.

c) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.

Example 6: (ex. 5, p. 174)

A car manufacturer keeps a database of all the cars that are available for sale at all the dealerships in Eastern Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both leather seats and a sunroof.

Example 7:

The probability that Dana will make the hockey team is 2/3. The probability that she will make the swimming team is 3/4. If the probability of Dana making both teams is 1/2, determine the probability that she will make:

a) at least one of the teams.

b) neither team.

Practice Questions:

P. 176 - 180, # 3, 4, 5, 6, 7, 8, 12, 15