Section 3.5: Conditional Probability (complete after Section 3.6)

In the previous section we determined the probability of 2 independent events by multiplying their individual probabilities.

We will determine the probability of 2 dependent events in a similiar way.

Dependent events:

Events whose outcomes are affected by each other.

Ex: 2 cards drawn from a deck, without replacement.

Conditional probability: P(B|A)

The probability of an event, B, occurring, given that another event, A, has already occurred.

NOTE: P(B|A) is NOT the same as B minus A, B\A.

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Using set notation, the formula is: $P(A \cap B) = P(A) \times P(B|A)$

Rearranging the formula for P(B|A) would give:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Example 1:

Cards are drawn from a standard deck of 52 cards (without replacement). Calculate the probability of obtaining:

a) a king, then another king

b) a club, then a heart

c) a black card, then a heart, then a diamond

Example 2:

A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. If Jocelyn draws 2 chips, at random, from a box of 100 chips, what is the probability that both of the chips will be defective?

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Example 3:

A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Example 4:

A hockey team has jerseys in three different colors. There are 4 green, 6 white and 5 orange jerseys in the hockey bag. Todd and Blake are given a jersey at random (without replacement). Students were asked to write an expression representing the probability that both jerseys are the same color. Which student correctly identified the probability and why?

Tony	$\left(\frac{2}{4}\right)\left(\frac{2}{6}\right)\left(\frac{2}{5}\right)$
Sam	$\left(\frac{2}{4}\right)+\left(\frac{2}{6}\right)+\left(\frac{2}{5}\right)$
Lesley	$\left(\frac{4}{15}\right)\left(\frac{3}{14}\right)+\left(\frac{6}{15}\right)\left(\frac{5}{14}\right)+\left(\frac{5}{15}\right)\left(\frac{4}{14}\right)$
Dana	$\left(\frac{4}{15}\right)\left(\frac{4}{15}\right)+\left(\frac{6}{15}\right)\left(\frac{6}{15}\right)+\left(\frac{5}{15}\right)\left(\frac{5}{15}\right)$

Example 5: (ex. 3, p. 185)

According to a survey, 91% of Canadians own a cellphone. Of theses people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

Example 6: (ex. 4, p. 186)

Hillary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on clam days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hillary's team will win tomorrow?

Example 7: (ex. 2, p. 184)

Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6, using each method below.

a) A Venn diagram

b) A formula

Practice Questions:

P. 188 - 191, # 1, 4, 7, 9, 10, 16, 18, 19