CHAPTER 4 Rational Expressions and Equations

Section 4.1: Equivalent Rational Expressions

Rational Expression:

A **rational expression** is any expression that can be written as the quotient of two polynomials, in the form $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$.

Rational Expressions include:

- A fraction that contains a polynomial in the numerator or denominator or both.
- A fraction that contains a variable.
- NOTE: All rational expressions are algebraic fractions, but not all algebraic fractions are rational expressions.

For example,

$$\frac{1}{x}, \quad \frac{m}{m+1}, \quad \frac{y^2-1}{y^2+2y+1}$$

$x^2 - 1$ is a rational expression with a denominator of 1.

Example 1:

Which of the following are rational expressions?

$$\frac{2x}{y}, \quad \frac{4}{5}, \quad \frac{x^2-4}{x+1}, \quad 2\pi, \quad \frac{x^2}{4}, \quad \sqrt{5}, \quad \frac{\sqrt{x}}{2y}$$

Non - Permissible Values (NPVs):

Values that make the denominator of a rational expression equal zero.

To determine the NPVs:

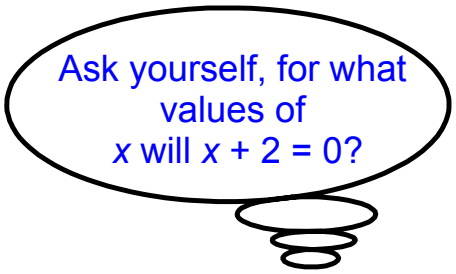
1) Set the denominator equal to zero.
2) Solve for the variable.

You may need to FACTOR to solve the equation.

All NPVs must be stated as **restrictions** on the variable in order to ensure the expression is defined.

Example 2:

Determine the non permissible values for the expression $\frac{x}{x+2}$ and then state the restrictions.



The non permissible value(s) for the expression are:

and the restrictions are:

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Example 3:

What are the non - permissible values of $\frac{x}{x^2-9}$?

Example 4:

Determine the non-permissible values for: $\frac{x-1}{3x^2-12x}$

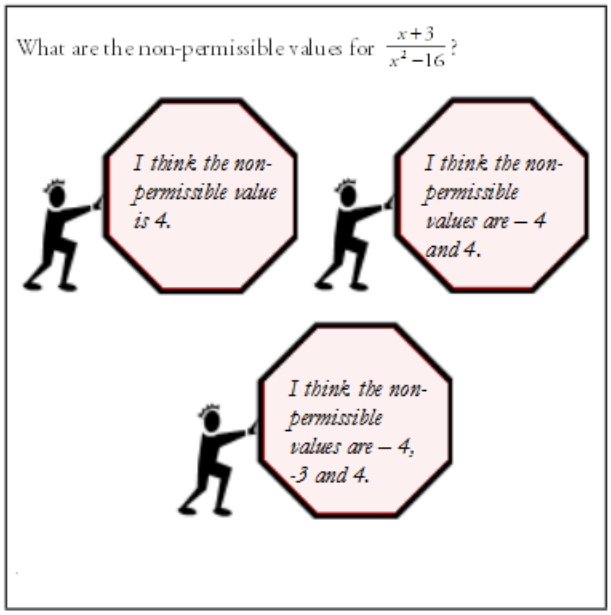
Your Turn: (ex. 3, p. 219)

Determine the NPVs for each rational expression and then state all the restrictions.

a) $\frac{4x^3}{6-2x}$

b) $\frac{-15}{x^3-4x}$

Example 5:



Who is correct? Justify your answer.

NOTE:

Non - permissible values and *inadmissable values* are not the same.

Non - permissible values are values that make the denominator of a rational expression zero.

Inadmissible values are values that do not make sense in a given context. For example, you cannot have a negative length.

Equivalent Rational Expressions:

Recall:

If we have a rational number and multiply/divide the numerator and denominator by a number (that is, if we multiply/divide the fraction by 1), it does not change the number.

We say the two fractions are ***equivalent rational numbers***.

For example, consider the fraction $\frac{3}{2}$.

$$\frac{3}{2} = \frac{3}{2}\times 1 = \frac{3}{2}\times\frac{4}{4} = \frac{12}{8}$$

We say $\frac{3}{2}$ and $\frac{12}{8}$ are equivalent fractions.

Does this apply to rational expressions as well?

Yes, it is similar but there are some restrictions!

Equivalent Rational Expressions:

Two rational expressions are equivalent only if they have the **same restrictions**.

This is accomplished by:

1. Multiplying or dividing the numerator and denominator by a number.
2. Multiplying the numerator and denominator by a factor that appears in the denominator.

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Example 6:

a) Are $\frac{7x}{x-2}$ and $\frac{14x}{2(x-2)}$ equivalent rational expressions?

b) Are $\frac{7x}{x-2}$ and $\frac{7x^2}{x(x-2)}$ equivalent rational expressions?

Example 7: (ex. 2, p. 218)

a) Write a rational number that is equivalent to $\frac{8}{12}$.

b) Write a rational expression that is equivalent to $\frac{4x^2+8x}{4x}$.

Example 8: (ex. 4, p. 220)

For each of the following, determine if the rational expressions are equivalent.

a) $\frac{9}{3x-1}$ *and* $\frac{-18}{2-6x}$

b) $\frac{2-2x}{4x}$ *and* $\frac{x-1}{2x}$

Practice Questions:
Worksheet 4.1,
p. 222 - 223, #9a(i, ii, iii), 11a(i, ii, iii), 16abcd