## Section 5.3 and 5.4: <br> Modelling Data with a Line/Curve of Best Fit

A line/curve of best fit is a straight line/curve that best approximates the trend in a scatter plot.


Regression results in an equation that balances the points in the scatter plot on both sides of the line/curve.

A line/curve of best fit can be used to predict values that are not recorded or plotted.

Predictions can be made, either within the data (interpolation) or outside the data (extrapolation), by reading values from the line/curve of best fit on a scatter plot of by using the equation of the line/curve of best fit.

When using a regression model, however, caution should be used in the range of the data since predicting values too far outside the data may be inappropriate and yield unrealistic answers.

## Example 1:

The winning times for the men's 20 km biathlon in the Winter Olympics from 1964 to 2010 (except for 2002) are shown in the table.

| Year | 1964 | 1968 | 1972 | 1976 | 1980 | 1984 | 1988 | 1992 | 1994 | 1998 | 2006 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Winning <br> Time <br> (min) | 80.4 | 73.8 | 75.9 | 74.2 | 68.3 | 71.9 | 56.6 | 57.6 | 57.4 | 56.2 | 54.3 | 48.4 |

One graphing technology that can be used to help determine an equation of the line or curve of best fit is a graphing calculator.

Here are a few screen shots from a graphing calculator to show the data.

a) Write the equation of the line of best fit.
b) Using the equation of the line of best fit, determine a possible winning time for the event in the 2002 Winter Olympics to the nearest hundredth of a minute.
c) Compare your estimate with the actual winning time of 51.0 min in 2002.

## Example 2:

Consider the data in the table.

| $\boldsymbol{x}$ | 0 | 2 | 4.5 | 5.2 | 9.5 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 5.1 | 6.7 | 8.2 | 8.8 | 11.9 | 13.4 |

Using technology, the equation of the line of best fit was determined to be:

$$
y=0.693 x+5.183
$$

a) Determine, to the nearest tenth, the value of $y$ when $x$ is 10.6 .
b) Determine, to the nearest tenth, the value of $x$ when $y$ is 9.8

## Example 3:

Matt buys T-shirts for a company that prints art on T-shirts and then resells them. When buying the T -shirts, the price Matt must pay is related to the size of the order. Five of Matt's past orders are listed in the table below.

| Number of Shirts | Cost per Shirt <br> (\$) |
| :---: | :---: |
| 500 | 3.25 |
| 700 | 1.95 |
| 200 | 5.20 |
| 460 | 3.51 |
| 740 | 1.69 |



Matt has misplaced the information from his supplier about price discounts on bulk orders. He would like to get the price per shirt below $\$ 1.50$ on his next order.

Creating a scatter plot and completing a linear regression, using technology, the equation for the linear regression function to model the data was determined to be:

$$
P=-0.0065 n+6.5
$$

a) What do the slope and $y$-intercept of the equation of the linear regression function represent in this context?
b) Use the linear regression function to extrapolate the size of order necessary to achieve the price of $\$ 1.50$ per shirt.

ASSIGN: p. 301, \#3, 4, 6, 9, 10

## Example 4:

Audrey is interested in how sped plays a role in car accidents. She knows that there is a relationship between the speed of a car and the distance needed to stop. She has found some experimental data (p.308) on a reputable website, and she would like to write a summary for the graduation class website.

Creating a scatter plot of this data she noticed a quadratic pattern and therefore completed a quadratic regression on the data which yielded the equation:

$$
y=0.008 x^{2}+0.539 x-10.449
$$

a) Use the equation to compare the stopping distance at $30 \mathrm{~km} / \mathrm{h}$ with the stopping distance at $50 \mathrm{~km} / \mathrm{h}$, to the nearest tenth of a meter.
b) Determine the maximum speed that a car should be travelling in order to stop within 4 m , the average length of a car.
(Note: Use the "trace" feature on a graphing calculator or read directly from a graph. Refer to p. 310 in the textbook for the graph.)

## Example 5:

The following table shows the average retail price of gasoline, per litre, for a selection of years in a 30-year period beginning in 1979.

| Years after 1979 | Price of Gas (c/L) | Years after 1979 | Price of Gas (¢/L) |
| :---: | :---: | :---: | :---: |
| 0 | 21.98 | 17 | 58.52 |
| 1 | 26.18 | 20 | 59.43 |
| 2 | 35.63 | 22 | 70.56 |
| 3 | 43.26 | 23 | 70.00 |
| 4 | 45.92 | 24 | 74.48 |
| 7 | 45.78 | 25 | 82.32 |
| 8 | 47.95 | 26 | 92.82 |
| 9 | 47.53 | 27 | 97.86 |
| 12 | 57.05 | 28 | 102.27 |
| 14 | 54.18 | 29 | 115.29 |



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a) Given the scatter plot, what polynomial function could be used to model the data? Explain.
b) The equation of the cubic regression function that models this data is $P=0.0123 n^{3}-0.4645 n^{2}+6.295 n+23.452$ where $P$ represents the average price of gas per litre and $n$ represents the number of years after 1979. Use this equation to estimate the average price of gas in 1984 and 1985.
c) Estimate the year in which the average price of gas was 56.0 cents/L.

## Gasoline Retail Price

ASSIGN: p. 313, \#2, 3abc, 4, 8abc


Extra Example: (if using graphing calculator)
The concentration (in milligrams per litre) of a medication in a patient's blood is measured as time passes. Susan has collected the following data and is attempting to express the concentration as a polynomial function of time.

| Time (T hours) | 0 | 1.5 | 3 | 4.5 | 6 | 7.5 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concentration (C mg/L) | 0 | 26.9 | 41.2 | 47.8 | 46.0 | 36.8 | 20.3 |

a) Use technology to create a scatter plot and determine a quadratic regression function that models the data. Round parameters $a, b$, and $c$ to two decimal places.
b) The doctor has decided that the patient needs a second dose of medication when the concentration in the blood is less than $10 \mathrm{mg} / \mathrm{L}$. If the first dose of medication was given at 9:00am, at what time should the second dose be given?

## Key Strokes for Graphing Calculator for Example 1




