Section 6.3: Solving Exponential Equations

Exponent Laws	***Examples:***		
1. Zero Exponent: $b^0 = 1$	a) $\left(\frac{2}{3}\right)^0$	b) $5x^0$	c) $(5x^2y^3)^0$
2. Negative Exponent: $b^{-x} = \frac{1}{b^x}$	a) 3^{-2}	b) $\left(\frac{3}{4}\right)^{-2}$	c) $\frac{1}{x^{-2}}$
3. Product Rule: $b^x \cdot b^y = b^{x+y}$	a) $2^2 \times 2^3$	b) $x^4 \bullet x^2$	c) $5x^2y^4(3x^3y^2)$

Exponent Laws	*Examples:*		
4. Quotient Rule: $\frac{b^x}{b^y} = b^{x-y}$	a) $\frac{x^5}{x^3}$	b) $\frac{12x^2}{4x^{-3}}$	c) $\frac{16x^3y^7}{8x^5y^4}$
5. Power Rule: $(b^x)^y = b^{xy}$	a) $\left(2^5\right)^2$	b) $\left(2x^{-2}\right)^3$	c) $\left(\frac{1}{3x^5}\right)^{-2}$
6. Rational Exponents: $b^{\frac{1}{n}} = \sqrt[n]{b}$; $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$	a) $9^{\frac{1}{2}}$	b) $64^{\frac{2}{3}}$	c) $\left[\left(\frac{16}{9}\right)^{-3}\right]^{\frac{1}{2}}$

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7. Common Base Rule: a) $2^x = 2^3$ b) $5^{x+3} = 5^4$

$b^x = b^y$ if and only if $x = y$

Example 1:

Express each of the following as a power with a base of 2.

a) 8 b) $\frac{1}{16}$ c) 8^{-2} d) $8^{\frac{2}{3}}\left(\sqrt{16}\right)^3$

Your Turn

Express each of the following as a power with a base of 3.

a) 27^2 b) $\left(\frac{1}{9}\right)^4$ c) $27^{\frac{2}{3}}\left(\sqrt[3]{81}\right)^6$

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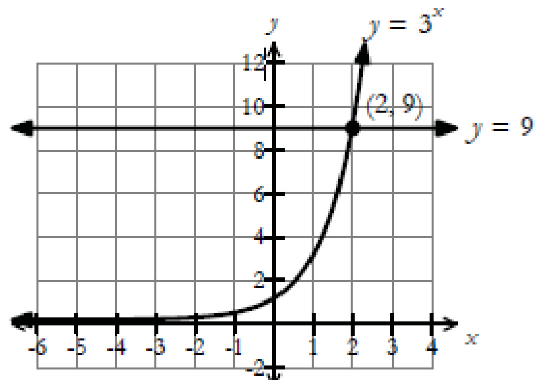
To Solve Exponential Equations:

- write both sides of the equation with the same base
- equate the exponents

Example 2:

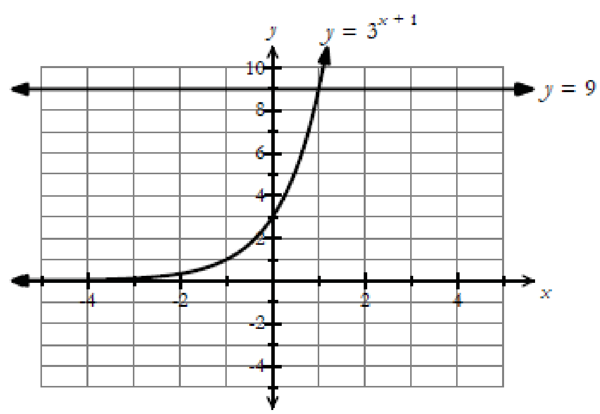
Solve for x: $3^x = 9$

The solution for the equation $3^x = 9$ can also be depicted graphically. We treat each side of the equation as 2 separate functions. The x-value of the point of intersection is the solution to the equation, $x = 2$.



Example 3:

a) Use the graph to determine the solution for $3^{x+1} = 9$.



b) Verify the solution algebraically.

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Example 4:

Solve each equation algebraically:

Rewrite each equation with the same base and equate the exponents.

a) $2^{x-1}=16$

b) $4^{2x}=8^{2x-3}$

c) $4(3^{x+2})=36$

d) $8^{3x-4}+7=71$

Your Turn:

Algebraically determine the solution for each of the following equations:

a) $3^{2x+1}=3^{x+2}$

b) $4^{3x+5}=2^{4x-3}$

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c) $3(2)^{3x-2} = 48$

d) $9(2^{3x+5}) - 8 = 28$

Example 5:

Solve each equation algebraically:

Fraction in the base

= Negative Exponent

a) $5^x = \frac{1}{125}$

b) $(32)^{x-2} = \left(\frac{1}{4}\right)^{5x-3}$

Your Turn:

a) $\left(\frac{1}{8}\right)^{x-3} = 16^{2x+1}$

b) $2(4)^{2x} = \frac{1}{32}$

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Example 6:

Solve each equation algebraically:

Radical = Fractional Exponent

a) $\sqrt{8} = 2^{3x-4}$

b) $5^{x+2} = \sqrt[3]{25}$

Your Turn:

a) $27^{2x-1} = \sqrt[3]{3^{2x}}$

b) $\sqrt{3^{x}} = 9^{2x+1}$

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Example 7:

Solve each equation algebraically:

a) $9^{x-1} \times 81^{2x-1} = 27^{3x-2}$

b) $\dfrac{64^{x-1}}{16^{2x+2}} = 2^{x-2}$

c) $5^{x^2+2x} = 125$

→

Your Turn:

a) $4^{3x+2} \times 32^{x-2} = 8^{3x-4}$

b) $\dfrac{125^{2x+1}}{625^{x+2}} = 3125^{x+2}$

Example 8: Identify and correct the error.

$$\sqrt{5} = 25^{3x+4}$$
$$5^{\frac{1}{2}} = 5^{2(3x+4)}$$
$$5^{\frac{1}{2}} = 5^{6x+4}$$
$$\frac{1}{2} = 6x+4$$
$$2 = 12x+8$$
$$-6 = 12x$$
$$-\frac{1}{2} = x$$

Practice:

p. 361, #2abcd, 4cdef, 5abc, 7bdf

Recall that an exponential expression arises when a quantity changes by the same factor for each unit of time.

For example,

- a population doubles every year;
- a bank account increases by 0.1% each month;
- a mass of radioactive substance decreases by 1/2 every 462 years.

The ***half-life*** exponential function is given by the equation:

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where $A(t)$ is the value after time, t

A_0 is the initial value

h is the **half-life**

The ***doubling*** exponential function is given by the equation:

$$A(t) = A_0(2)^{\frac{t}{d}}$$

where $A(t)$ is the value after time, t

A_0 is the initial value

d is the **doubling time**

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Example 9:

The half-life of a radioactive isotope is 30 hours. The amount of radioactive isotope A(t), at time t, can be modelled by the function:

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Algebraically determine how long it will take for a sample of 1792 mg to decay to 56 mg.

Solution:

Example 10:

The population of trout growing in a lake can be modeled by the function $P(t) = 200(2)^{\frac{t}{5}}$ where *P*(*t*) represents the number of trout and *t* represents the time in years after the initial count. How long will it take for there to be 6400 trout?

Note:
- the value of 200 represents the initial number of trout
- the number of trout doubles every 5 years

Solution:

Your Turn:

The half life of Radon 222 is 92 hours. From an initial sample of 48g, how long would it take to decay to 6g?

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

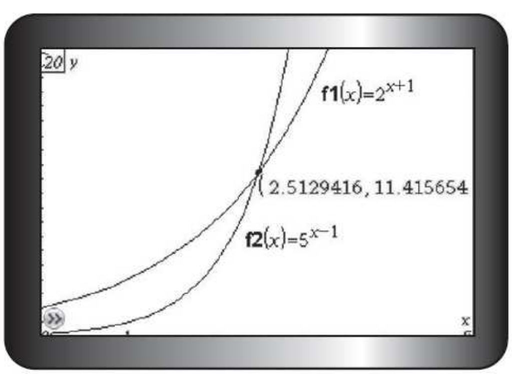
Example 11: (ex. 4, p. 359)

Solve:

$$2^{x+1} = 5^{x-1}$$

Since neither base can be written as a power of the other, we can't equate the exponents. We will learn how to solve this algebraically in Unit 7. For now, we will solve by using graphing technology.

$y_1 = 2^{(x+1)}$ $\quad$ $y_2 = 5^{(x-1)}$



Solution: x ~ 2.5

Practice:

p. 363 - 365, #11a, 15, 16b

+ worksheet