Section 6.4: Modelling Data Using Exponential Functions

In the previous unit, linear, quadratic and cubic regressions were performed on polynomial functions. Similarly, technology can be used to create a scatter plot and determine the equation of the exponential regression function that models the data.

Example 1: (ex. 1, p. 371)

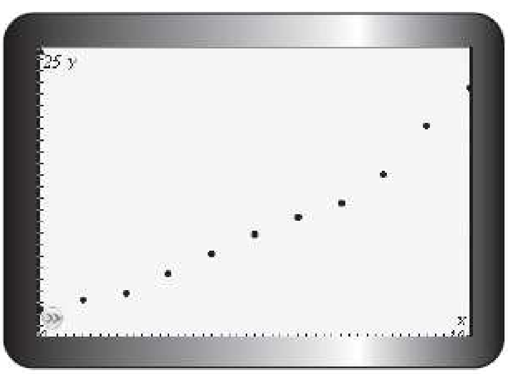
The population of Canada from 1871 to 1971 is shown in the table below. In the third column, the values have been rounded.

Year	Actual Population of Canada	Population of Canada (millions)
1871	2 436 297	2.44
1881	3 229 633	3.23
1891	3 737 257	3.74
1901	5 418 663	5.42
1911	7 221 662	7.22
1921	8 800 249	8.80
1931	10 376 379	10.38
1941	11 506 655	11.51
1951	14 009 429	14.01
1961	18 238 247	18.24
1971	21 568 305	21.57

Statistics Canada

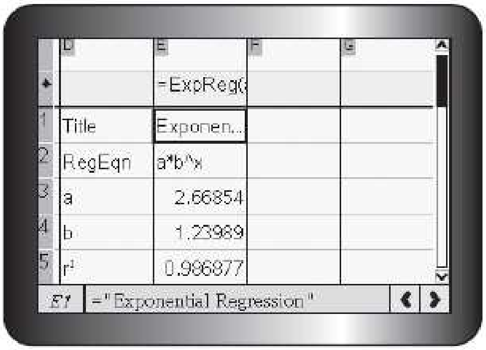
a) Using graphing technology, create a graphical model and an algebraic exponential model for the data.

Let x represent the number of 10-year intervals since 1871. Let y represent the population of Canada in millions.

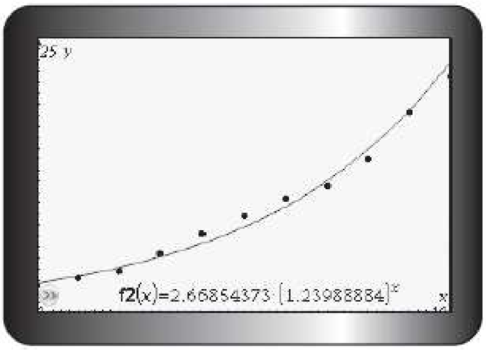


As the x-values get larger, the y-values also grow larger, but not at a constant rate.

The data can be modelled by an ***exponential growth function.***



An exponential regression on the data determines the equation of the curve of best fit.



The equation is verified by graphing it on the same grid as the given points.

The exponential equation that models the data is:

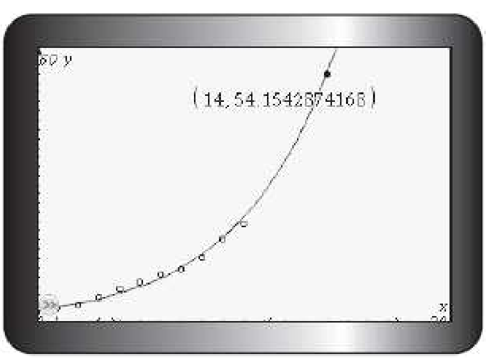
$$y = a(b)^x$$
$$y = 2.67(1.24)^x$$

b) Assuming that the population growth continued at the same rate to 2011, estimate the population in 2011. Round your answer to the nearest million.

Since x represents the number of 10-year intervals since 1871,

$$x = \frac{2011 - 1871}{10} = 14$$

$$y = 2.67(1.24)^x$$
$$y = 2.67(1.24)^{14}$$
$$y = 54.25$$
$$y = 54 \text{ million}$$



The solution can also be *extrapolated* from the graph.

Example 2: (p. 373)

Sonja did an experiment to determine the cooling curve of water. She placed the same volume of hot water in three identical cups. Then she recorded the temperature of the water in each cup as it cooled over time. Her data for three trials is given as follows.

Trial 1

Time (min)	Temperature (°C)
0	80
5	69
10	61
20	45
30	34
40	26

Trial 2

Time (min)	Temperature (°C)
0	75
5	66
10	59
20	44
30	32
40	23

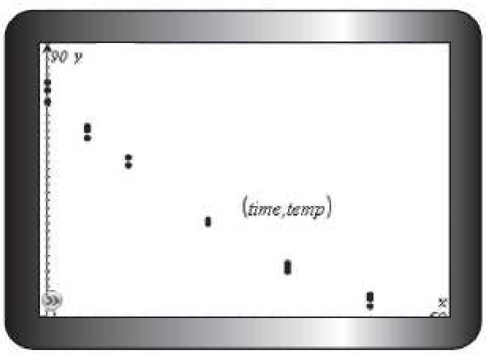
Trial 3

Time (min)	Temperature (°C)
0	78
5	68
10	61
20	44
30	33
40	25

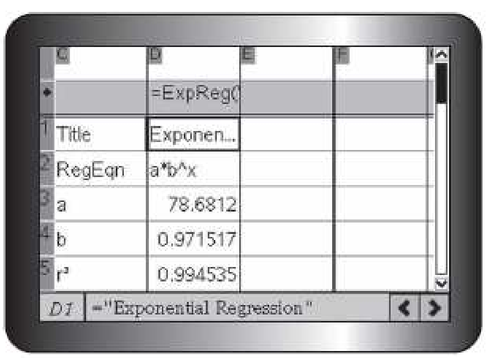
a) Construct a scatter plot to display the data. Determine the equation of the exponential regression function that models Sonja's data.

Let x represent the time, in minutes, since the experiment began.

Let y represent the temperature in degrees Celsius.

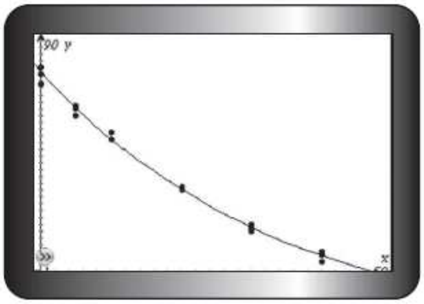


The data can be modelled by an ***exponential decay function.***



An exponential regression on the data determines the equation of the curve of best fit.

Note: The initial temperatures of the three samples were not the same: 80 °C, 75 °C, and 78 °C.
The regression model defines a = 78.68, which is close to all three initial values.



The equation is verified by graphing it on the same grid as the given points.

The exponential equation that models the data is:

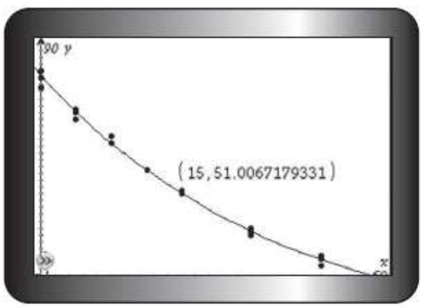
$$y = a(b)^x$$
$$y = 78.68(0.97)^x$$

The solution can also be *extrapolated* from the graph.

b) Estimate the temperature of the water 15 min after the experiment began. Round your answer to the nearest degree.

After 15 minutes, $y = 78.68(0.97)^{15}$

$$y = 50^0 C$$

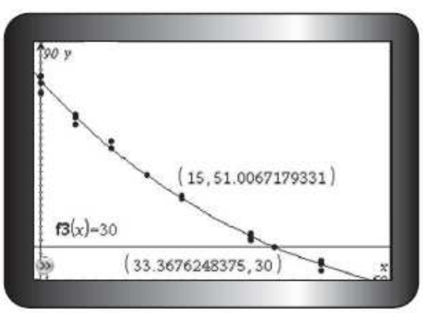


The solution can also be *interpolated* from the graph.

c) Estimate when the water reached a temperature of 30 °C. Round your answer to the nearest minute.

$$y = 78.68(0.97)^x$$

$$30 = 78.68(0.97)^x$$



The point of intersection is (33.367..., 30),

so x = 33 minutes.

Practice Questions:

p. 377-382, #4c, 5c, 8c
(use the graphs and regression equations from p. 740)