Section 6.5: Financial Applications Involving Exponential Functions

1. Simple Interest:

Simple interest is calculated only in terms of the original amount invested, not on the accumulated interest.

Simple Interest Formula: $A = P + Prt$ or $A = P(1 + rt)$

where P is the principal amount

t is the time in years

r is the interest rate per annum (as a decimal)

NOTE: A is the sum of the principal (P) and the accumulated interest (Prt)

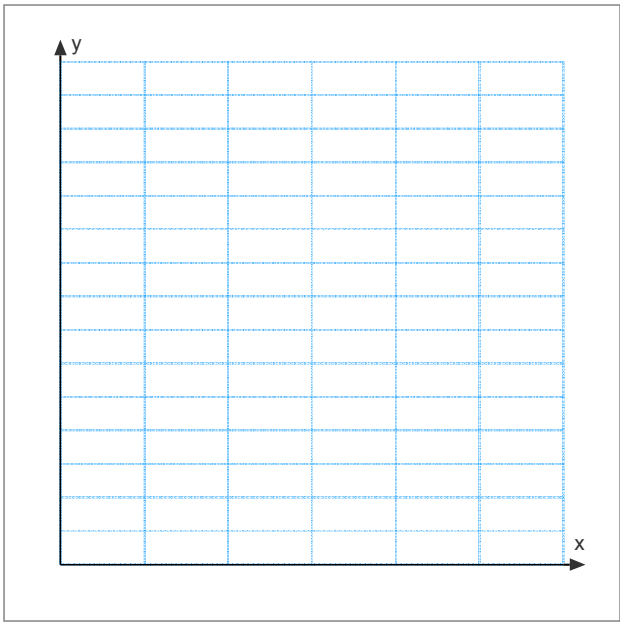
Example 1:

Kyle invested his summer earnings of $5000.00 at 8% simple interest, paid annually.

a) Create a table of values and graph the growth of the investment for 6 years using time, in years, as the domain and the value of the investment as the range.

Time (years)	Value of Investment ($)
0	
1	
2	
3	
4	
5	
6	

a) What does the shape of the graph tell you about the type of growth?



b) Why is the data discrete?

c) What do the y-intercept and slope represent for the investment?

d) What is the value of the investment after 10 years?

e) How much interest was earned after 10 years?

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2. Compound Interest:

Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest.

Compound Interest Formula: $A = P(1+i)^n$

where A is the future value

P is the principal amount

i is the interest rate **per compounding period** (expressed as a decimal)

t is the time in years

n is the **number of compounding periods**

n is NOT the number of years!

Refer to previous example of \$5000.00 (P) in a savings account earning an annual interest of 8%.

Time (years)	Amount of Annual Interest	Value of Investment (\$)
0		
1		
2		
3		

NOTE: The accumulated interest and the value of the investment do not grow by a constant amount as they do with simple interest.

An exponential regression to model the investment would result in the equation: $y = 5000(1.08)^x$

Note how this compares to: $A = P(1+i)^n$.

Investments can also have daily, weekly, monthly, quarterly, semi-annually, or annually compounding periods.

Compounding Period	Number of Times Interest is Paid	Interest Rate per Compounding Period, i
Daily	365 times per year	$i = \frac{\text{annual rate}}{365}$
Weekly	52 times per year	$i = \frac{\text{annual rate}}{52}$
Bi-Weekly	26 times per year	$i = \frac{\text{annual rate}}{26}$
Semi-monthly	24 times per year	$i = \frac{\text{annual rate}}{24}$
Monthly	12 times per year	$i = \frac{\text{annual rate}}{12}$
Quarterly	4 times per year	$i = \frac{\text{annual rate}}{4}$
Semi-annually	2 times per year	$i = \frac{\text{annual rate}}{2}$
Annually	1 time per year	$i = \frac{\text{annual rate}}{1}$

For example, if \$5000 is invested at 6% compounded monthly,

$$i = \frac{\text{annual rate}}{12} = \frac{0.06}{12} = 0.005$$

The compound interest formula is defined as:

$$A = 5000(1.005)^n$$

where n is the number of months, NOT the number of years

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Example 2: Complete the table if the interest rate is 4.8% per year.

Compounding Period	Number of Times Interest is Paid	Interest Rate per Compounding Period (i)
Bi-Monthly		
Monthly		
Quarterly		
Semi-Annually		
Annually		

Example 3:

If \$5000 is invested, calculate A (the future value) using $A = P(1+i)^n$ for each situation.

a) 11% per year, compounded quarterly for 3 years

b) 6.5% per year, compounded semi-annually for 3 years

c) 15.6% per year, compounded monthly for 2 years

Example 4:

$3000 was invested at 6% per year compounded monthly.

a) Write the exponential function in the form: $A = P(1+i)^n$

b) What will be the future value of the investment after 4 years?

Example 5:

An automobile that originally costs $24 000 loses one-fifth of its value each year. What is the value after 6 years?

Example 6:

$2000 is invested for 3 years at an annual interest rate of 9% compounded monthly. Lucas solved the following equation:

$$A = 2000(1.0075)^3$$

Correct the error and solve the problem.

Example 7:

Nora is about to invest $5000 in an account that pays 6% interest a year compounded monthly for the next 3 years. A different financial institution offers 6.5% interest a year compounded semi-annually for the next 3 years. Write a function that models the growth of Nora's investment for each situation. Should Nora invest her money in this financial institution instead? Explain why or why not.

Practice Questions:

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