Section 7.2 Evaluating Logarithmic Expressions

A logarithmic function can be expressed as an exponential function and vice versa.

$$\boxed{y = \log_b x \longleftrightarrow x = b^y}$$

Logarithm = Exponent

The expression $y = \log_b x$ means "the exponent that must be applied to base b to give the value of x.

for example, $\log_2 8 = 3$ since $2^3 = 8$

Two Specific Types of Logarithms:

1. Common Logarithm: $y = \log_{10} x \longleftrightarrow x = 10^y$
   - Base 10

   or

   $y = \log x$

2. Natural Loagarithm: $y = \log_e x \longleftrightarrow x = e^y$
   - Base e

   or

   $y = \ln x$

NOTE:
One way to convert from Exponential form to Logarithmic form is to remember:

$$Base^{Exponent} = Number \longleftrightarrow \log_{Base} Number = Exponent$$

$B^E = N$ the $\log_B N = E$

"Ben the Log Bunny"

$\longrightarrow$

Example 1:
Convert the following to **exponential form**.

a) $y = \log_3 x$ b) $y = \log x$ c) $y = \ln x$

Example 2:
Convert the following to **logarithmic form**.

a) $x = 10^y$ b) $x = 5^y$ c) $x = e^y$

Evaluating a Logarithmic Function

Example 3: Evaluate.

Ask yourself,

2 to what exponent is 8

$\log_2 8$

$y = \log_2 8$ Set the logarithmic function equal to y

$2^y = 8$ Write in exponential form

$2^y = 2^3$ Rewrite using common bases

$y = 3$ Since the bases are the same, the exponents must be the same.

Example 4: Evaluate.

a) $\log_3 81$

b) $\log_2 16$

c) $\log_4 64$

d) $\log_8 1$

e) $\log_7 7$

f) $\log_2(-4)$

Example 5: Evaluate.

a) $\log_3\left(\frac{1}{27}\right)$

b) $\log_{\left(\frac{1}{4}\right)} 64$

c) $\log_2 \sqrt{8}$

d) $\log_9 \sqrt[5]{81}$

Example 6: Which expression has the greater value?

a) $\log_2 1 + \log_2\left(\frac{1}{8}\right)$

b) $\log_{\frac{1}{2}} 16 - \log_{\frac{1}{3}} 27$

Example 7:

Evaluate each using a calculator to two decimal places.

a) $\log 85$ b) $\ln 23$

Example 8: (Ex. 2, p. 429)

Determine the value of y in each exponential equation.

a) $81 = 10^y$ b) $25 = e^y$

Example 9:

Mary evaluated $\log(-3.24)$ on her calculator and an error message was displayed. Explain why an error message occurred.

Practice: p. 436, #5 - 13

Solving Problems Involving Logarithmic Scales

Many real life situations have values that vary greatly. A logarithmic scale with powers of 10 can be used to make comparisons between large and small values more manageable.

Three examples of logarithmic scales are:

1. The Richter scale - used to measure the magnitude of an earthquake.
2. The pH scale - used to measure the acidity of a solution.
3. The Decibel scale - used to measure sound level.

Example 1: See p. 433 for pH scale; 0 (acidic) - 14 (basic); pH = 7 is neutral

The pH, p(x), of a solution can be determined using the formula $p(x) = -\log x$, where the concentration of hydrogen ions, x, is measured in mol/L.

a) The hydrogen ion concentration of the solution is 0.0001 mol/L.

Calculate the pH of the solution.

b) Calculate the hydrogen ion concentration of lemon juice (pH = 2).

c) How many time more acidic is Solution A, with a pH of 1.6, than Solution B, with a pH of 2.5? Round your answer to the nearest tenth.

Example 2:

The magnitude of an earthquake, y, can be determined using $y = \log x$ where x is the amplitude of the vibrations measured using a seismograph. An increase of one unit in magnitude results in a 10-fold increase in the amplitude. Answer the following questions using the table below.

Location	Magnitude
Chernobyl, 1987	4
Haiti, January 12, 2012	7
Northern Italy, May 20, 2012	6

a) How many times as intense was the earthquake in Haiti compared to the one in Chernobyl?

b) How many times as intense was the earthquake in Haiti compared to the one in Northern Italy?

c) If a recent earthquake was half as intense as the one in Haiti, what would be the approximate magnitude?

Example 3:

Sound levels are measured in decibels. The decibel scale is logarithmic and is defined by the equation $\beta = 10(\log I + 12)$ where β is the sound level in decibels, db, and I is the sound intensity in watts per square metre, W/m^2. What is the sound level, to the nearest decibel, of each sound?

a) a conversation at 50 cm, if $I = 2 \times 10^{-4} \ W/m^2$ ☆

b) rustle of leaves, if $I = 1 \times 10^{-11} \ W/m^2$

c) siren at 30 m, if $I = 1 \times 10^{-2} \ W/m^2$

Practice:
p. 437- 438, #14bde, 15ace, 16ab,17, 19, 20