Section 7.4: Solving Exponential Equations using Logarithms

Recall in Unit 6 we solved exponential equations by writing with the same base and then equating the exponents.

For example,
$$2^x = 8$$
$$2^x = 2^3$$
$$x = 3$$

The same equation can be solved using logarithms.

> You can solve an exponential equation by taking the logarithms of both sides of the equation.

For example,
$$2^x = 8$$
$$\log 2^x = \log 8 \quad \textbf{(Take the logarithm of both sides)}$$
$$x \log 2 = \log 8 \quad \textbf{(Apply the power law)}$$
$$x = \frac{\log 8}{\log 2}$$
$$x = 3$$

Example 1: Solve the following, $2^x = 7$

> NOTE: This example cannot be solved by writing with the same base. It has to be solved using the "new" way!

Example 2:

a) Evaluate each of the following:

i) $\log_2 8$

ii) $\log_2 16$

b) Based on those answers, estimate $\log_2 9$.

c) Evaluate: $\log_2 9$

Example 3: Evaluate $\log_2 100$ to three decimal places.

Change of base formula:

$$\log_b x = \frac{\log x}{\log b}$$

Example 4: Evaluate to 3 decimal places.

a) $\log_4 120$

b) $\log_3 15$

Example 5: Solve each of the following

a) $2^{x+1} = 32$

i) Using common bases

ii) Using Logarithms

b) $3^{x-1} = 20$

c) $4(3)^{x} = 24$

d) $2^{x-1} = 3^{x+1}$

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Your Turn:

Solve for x: $2^{x+1} = 5^{x-1}$ ☆

Practice:
p. 455 - 458, #1ace,2ab,3ab,5ab,6ac,16

Word Problems:

In the last unit we completed questions involving half-life, doubling life, compound interest, and depreciation where we could solve the exponential equations by writing with the same base. We will now revisit those questions with one difference - we will not be able to write with the same base therefore we will have to take the log of both sides to finish solving the problem.

Example 6:

If a $1000 deposit is made at a bank that pays 12% per year, compounded annually, how long will it take for the investment to reach $2000.

Example 7: (ex. 3, p. 451)

Jahmal works in a laboratory that uses radioactive substances. The laboratory received a shipment of 500 g of radioactive radon-222. Only 13.417 g of the radon-222 remained 19.0 days later. Determine the half-life of radon-222 algebraically using logarithms.

Recall: The half-life equation is: $A = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$

Example 8: Error Analysis #8, p. 457

Dave thought that he could also solve the exponential equation in Example 1 by taking the logarithm of each side in the first step. However, he made an error in his solution. Correct Dave's error, and complete his solution.

Dave's Solution

$A = P(1 + i)^n$ $P = 3215$ $i = 0.024$ $A = 5000$	I substituted the given values into the compound interest formula.
The number of compounding periods, n, is unknown.	
$5000 = 3215(1.024)^n$	
$\log 5000 = \log (3215(1.024)^n)$	I took the common logarithm of each side of the equation.
$\log 5000 = n \log (3215(1.024))$	I used the power law of logarithms to rewrite the equation.
$n = \dfrac{\log 5000}{\log (3215(1.024))}$	I isolated n.
$n = 1.051...$	
It will take 2 years for the balance to reach $5000.	The interest is compounded annually, so I rounded up to 2 years. This answer is different from my first answer, and it seems much too small.

Practice:
p. 457, #10, 11, 12, 13 + Worksheet